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## LUNAR CRATER STATISTICS: THE HIGHLAND REGIONS

Code 1

by

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## Abstract

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The crater diameter (D) vs. cumulative frequency of occurrence (N) for lunar craters can be described by equations of the form

$$N = AD^B$$

with the constants A, B empirically determined from actual counts of craters. In this paper crater counts for highland regions in the Boston University Catalog of Lunar Craters have been used to determine the constants. It is found, however, that a single set of constants does not adequately describe the cumulative distribution function. Two sets of constants are required, their average value for a normalized area ( $10^6$  km) being:

	A	B
D > 40 Km	30,350	-2.392
D < 40 Km	9,035	-1.263

Previous statistical descriptions of the highland regions are shown to be seriously inadequate.

It is found that the most logical manner of rectifying the two-segment function is to assume that some very large craters have not been included in the crater counts. This leads to an hypothetical rectified cumulative distribution function with constants (normalized to  $10^6$  Km<sup>2</sup>) of

$$A = 8,819$$

$$B = -1.2269$$

Any theory of lunar surface formations should be able to account for the two-segment cumulative distribution function, or for a transition from an initial one-segment function to the two-segment function.

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## Introduction

The most distinctive feature of the surface of the moon, when viewed with even small magnification, is the abundance of craters. These craters were observed first by Galileo Galilei in 1609 and many observers since then have observed, named, and cataloged the visible craters. The early astronomers thought the craters to be volcanic domes because of a certain resemblance to terrestrial volcanoes. Current thought, however, holds that the craters are entirely, or almost so, the result of explosive impacts of meteors. (1)

Despite the long standing interest in lunar craters, only recently have steps been taken to gather the systematic data needed to explain their origins. Detailed and accurate counts of crater size versus frequency of occurrence have been made only in the last 5 years. Such counts even now are not fully satisfactory. The uses of statistics of crater size and frequency are threefold. First, by themselves they provide an additional and useful description of the lunar surface. Second, they provide a basis for extrapolation of known surface features. This extrapolation, together with data from radar and other means, will be vital in guiding lunar exploration in the next few years. And finally, any theory of crater formation must be tested against the statistics of the craters to see if it can account for the observed surface features.

There are two distinct types of lunar surface visible through telescopes and on photographs. These are called highlands and maria because of their visual appearance. The maria are regions with relatively flat surfaces, few mountain ranges, and generally sharply defined craters. The highlands contain mountain ranges and the larger craters are often somewhat indistinct, the walls being broken down in many places. There are many more craters in the highlands regions than in the maria. The crater statistics for the highlands are quite distinct from those for the maria. This report will deal only with the statistics of the highland areas.

#### The Statistical Methods

Statistics are developed for those highland areas included in the area of the Boston University Catalog of Lunar Craters. Specifically the area used is that area surveyed in Catalogs I, II and III.<sup>(2)</sup> This area is contained in plates C5, C6, and C7 of Kuiper's Photographic Atlas of the Moon.<sup>(3)</sup> The total area surveyed is  $.68 \times 10^6$  square miles, or 2 percent of the moon's surface. This accounts for about 3 percent of the highland area and includes almost 2000 craters of diameter greater than 1 kilometer.

Previous authors (Opik<sup>(4)</sup>, Shoemaker<sup>(5)</sup>, McGillem and Miller<sup>(6)</sup>) have observed that lunar craters obey a diameter-

frequency relationship of the form

$$N = AD^B \quad (1)$$

where  $N$  is the number of craters larger than a given diameter  $D$ , and  $A, B$  are constants determined from actual crater counts. We call this the cumulative distribution function for lunar craters. It is also often referred to in the form

$$\log N = \log A + B \log D \quad (2)$$

Equation (1) gives a straight line when plotted on full logarithmic graph paper, while equation (2) yields a straight line on rectilinear graph paper.

Differentiation of equation (1) with respect to diameter gives us the crater density function

$$P = \frac{dN}{dD} = B A D^{B-1} \quad (3)$$

or alternately, in the logarithmic form,

$$\log P = \log \left( \frac{dN}{dD} \right) = \log B + \log A + (B-1) \log D \quad (4)$$

where  $P$  is the crater density. The number of craters having diameters between  $D$  and  $D + dD$  is then

$$\text{Number} = P(D) \cdot dD = B A D^{B-1} dD . \quad (5)$$

P is called the density distribution function for lunar craters, and D , in equations (1) and (2), is the cumulative distribution function.

The density distribution function (3) or (4) is usually the more convenient of the functions to use in any mathematical analysis of lunar craters (e.g., predicting theoretical intersection frequencies for craters.<sup>(7)</sup>) However, actual crater counts to determine A and B must deal with finite samples yielding integer numbers. These numbers are often small (particularly at large diameters) resulting in a good deal of statistical fluctuation from one sample area to another. If craters are regarded as being randomly scattered over the lunar surface, then the expected statistical deviation of any given counted number of craters M will be proportional to  $\sqrt{M}$  . By fitting the cumulative distribution function to the data we can cause N to increase rapidly, thus decreasing the relative error. This amounts to a smoothing the statistical fluctuations of the data by integration, and the density distribution function can then be obtained by differentiating the smoothed function.

While smoothing the crater counts by integration has the advantage of minimizing statistical fluctuations, there

are two inherent difficulties in the method. The first difficulty is that significant fluctuations in density may be hard to recognize because of the smoothing. The second is that any error in the count at large diameters (and hence small cumulative numbers) will be carried through all subsequent data points for smaller diameters. When fitting the data to a density distribution function some local smoothing is still required because of the finite number of craters counted. For example, 2 craters might be found at 84 km diameter, 0 at 85 km, and 1 at 86 km diameter. We would not then want to say that  $P(D)$  is 0 at  $D = 85$ . The scheme of smoothing used is discussed below, in "Results."

In this study the Boston University punched card catalog of lunar craters has been ordered by diameter and crater counts made from these cards. The data has been fitted to both cumulative and density distribution functions by the method of least squares. The actual fitting was accomplished by means of a computer program written for an IBM 7094 computing system, and the fitting was made to the logarithmic form in both cases.

The program calculates the constants  $A, B$ , plus an estimate of the accuracy of  $B$  using Student's  $t$  test at a 99% confidence level. A backsolution is also performed on the data points, so that the estimated crater counts, together with their estimated statistical errors, can be compared with actual counts. The statistical methods employed

are discussed more fully in most texts on statistics or regression analysis. See, for example, Goulden<sup>(8)</sup>, Methods of Regression Analysis, Chapter 5. A sample of the output from the computer is shown in figure 1.

Figure 1

C-5 Normalized

$$Y = -.15830492E + 01 X + .42915931E + 01 \underline{+} .23370014E-00$$

X	Y	XL	YL	YLE	YLD
4.000E-00	1.861E+03	6.02059E-01	3.26986E-00	3.33850E-00	1.186E-01
8.000E-00	6.139E+02	9.03089E-01	2.78815E-00	2.86195E-00	1.292E-01

X = Diameter

Y = Cumulative Number/Density

XL =  $\text{Log}_{10} X$

YL =  $\text{Log}_{10} Y$

YLE = Estimated YL

YLD = Estimated Statistical error in YLE

+ = Estimated error in coefficient B

## Results

The data obtained from crater counts of areas on plates C5, C6 and C7 are displayed directly in figures 2, 3 and 4 respectively. Crater counts have been normalized to an area of  $10^6 \text{ Km}^2$ , with the un-normalized counts listed in Appendix A. Immediately noticeable is the fact that the points do not form one straight line, but seem to form two straight lines which intersect between 35 and 50 km diameter.

When a single equation of the form (1) or (2) is fitted to these sets of data, the constants A,B given in Table I are obtained. This equation is the dashed line in figures 2, 3, 4.

Table I

Area	A	B	Dev*
C5	19,570	-1.5831	.13391
C6	19,630	-1.5889	.27311
C7	13,675	-1.4527	.04461

\*Sum of log deviations squared, i.e.,

$$\text{dev} = \sum_D (\log N \text{ actual} - \log A - B \log D)^2$$

Two sets of constants A,B can be obtained for each plate, however, by fitting the cumulative distribution function (1) or (2) to each line segment separately. The results of a two-segment fit of the equation are given in Table II, and the seg-

ments are the solid lines in figures 2, 3, 4.

Table II

Area	Large Diameter		Small Diameter		Dev*
	A	B	A	B	
C5	19,603	-2.7354	11,640	-1.3757	.02877
C6	1,456,186	-2.6973	7,185	-1.1768	.01581
C7	41,105	-1.7420	8,280	-1.2370	.00851

\*See Table I.

The density distribution data is displayed in Figures 5, 6 and 7. To obtain the measured density at each crater diameter, local smoothing has been employed. For densities in the diameter range 20 - 60 km, densities are averaged over a 10 km increment centered on the given diameter. For craters in the diameter range 11 - 15 km, a 3 km increment is used. Craters in the range 3 - 10 km use a 1 km averaging increment. Then, in all cases,  $\text{density} = (\text{craters})/(\text{increment})$ . Densities for diameters greater than 50 km have not been used, because the number of craters in this size range is too small to give meaningful numbers.

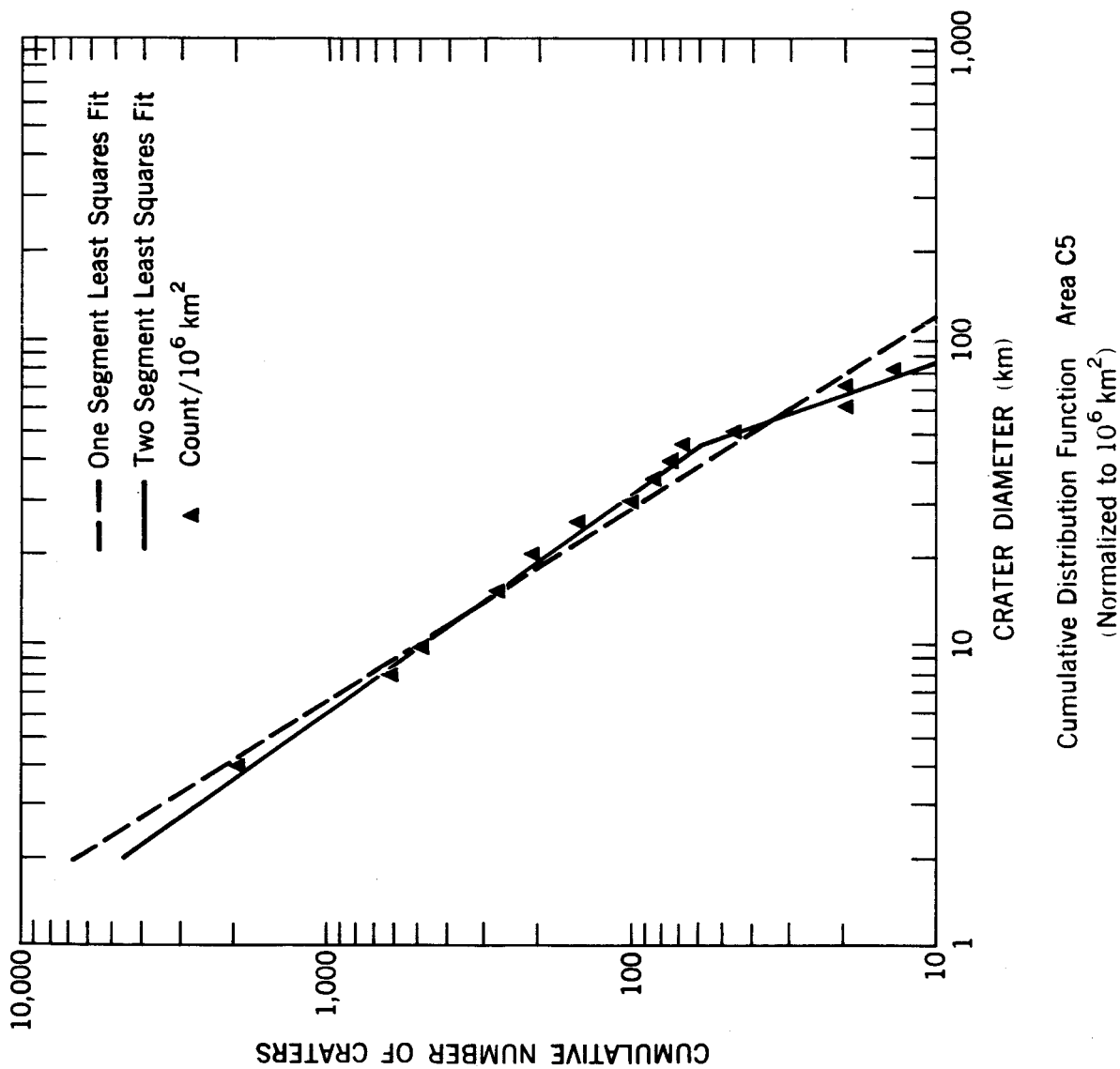
9-a

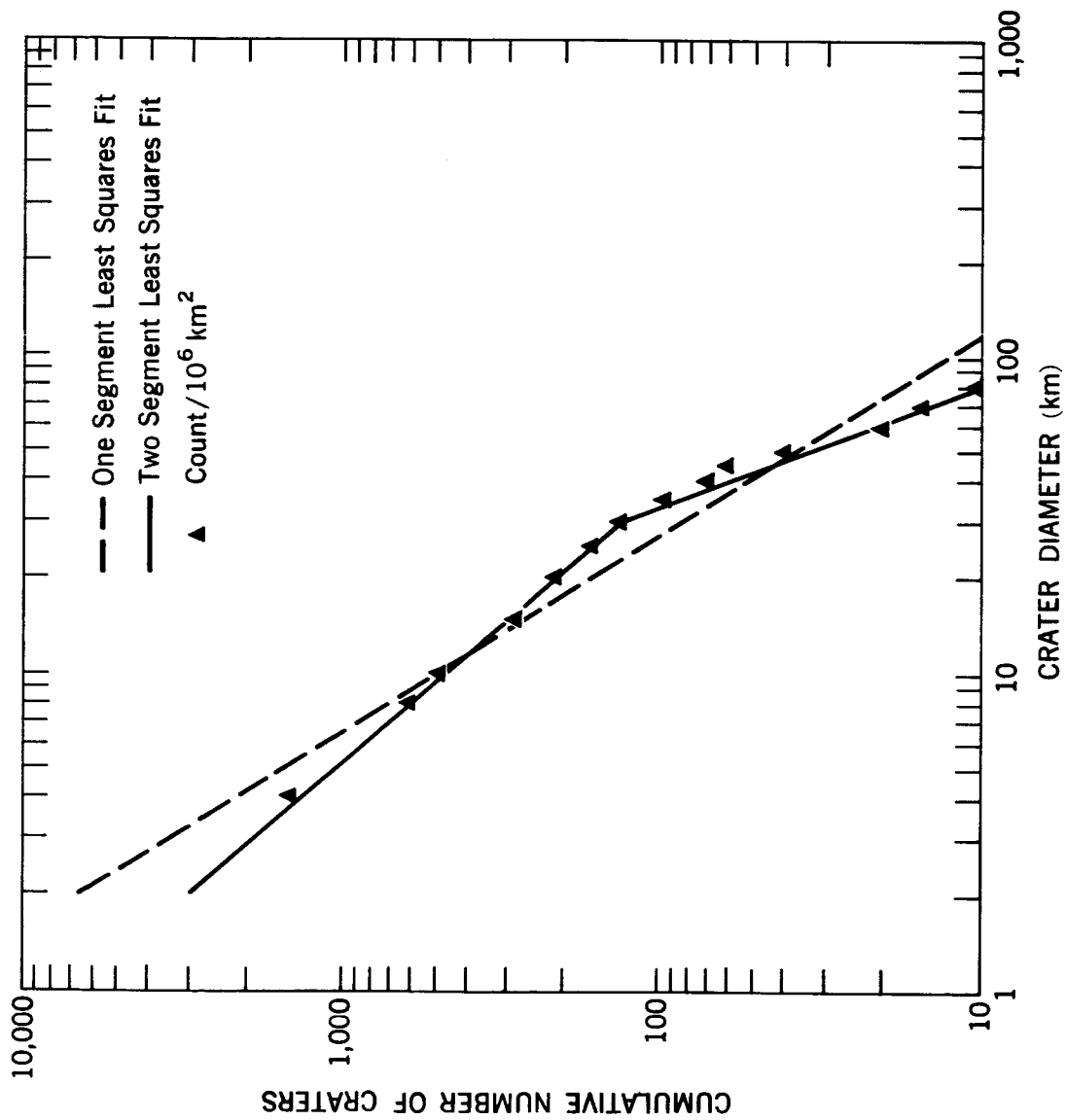
**Cumulative Distribution Functions**

**Figure 2:           Area           C5**

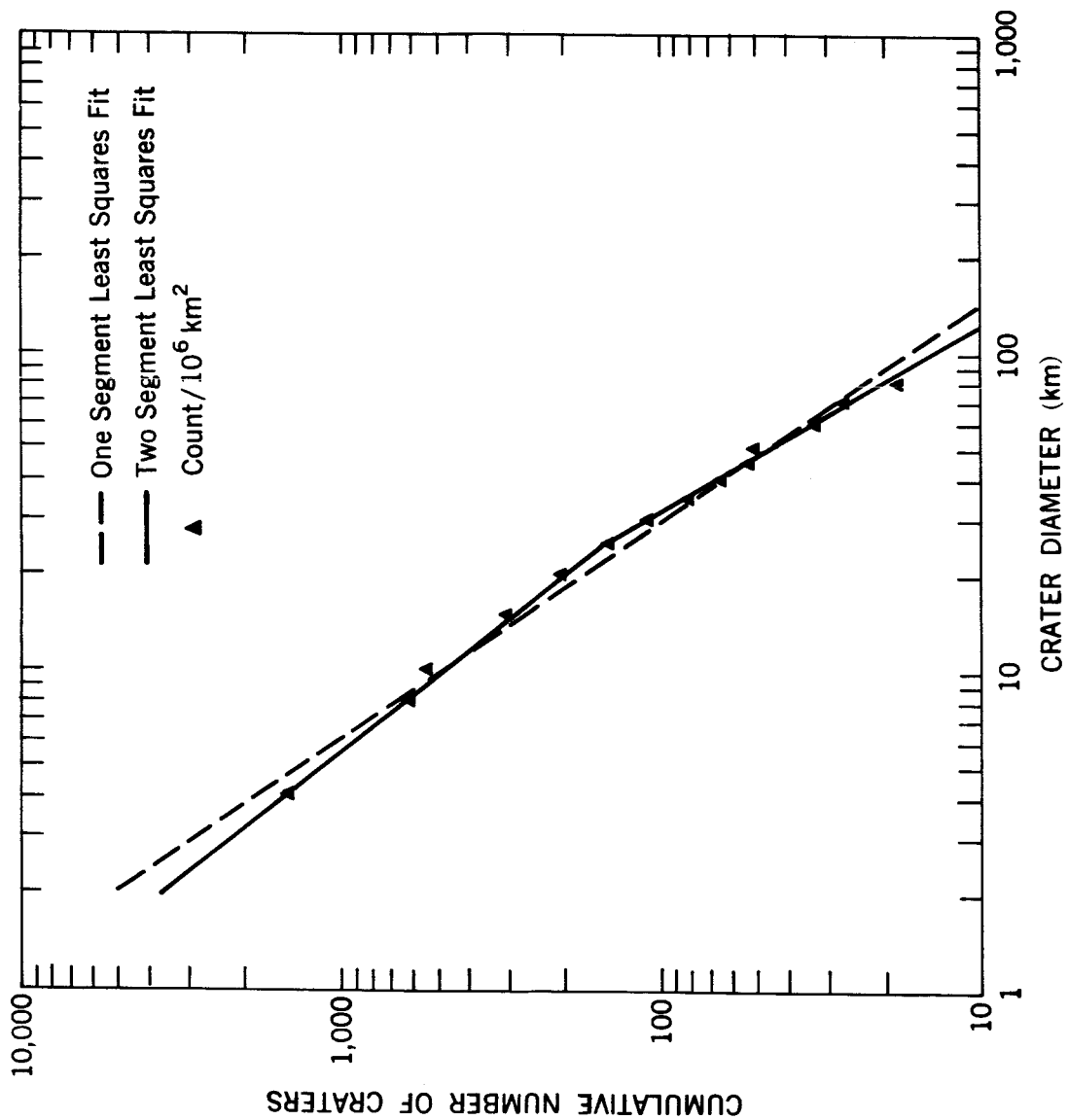
**Figure 3:           Area           C6**

**Figure 4:           Area           C7**





Cumulative Distribution Function—Area C6  
(Normalized to  $10^6 \text{ km}^2$ )



Cumulative Distribution Function—Area C7  
(Normalized to  $10^6 \text{ km}^2$ )

TABLE III

Density	Distribution	Function
Area	B - 1	B
C5	-2.2842	-1.2842
C6	-1.9612	-0.9612
C7	-2.2481	-1.2481

The density functions seem to exhibit "fine-structure", particularly toward the smaller diameters, as can be seen in figures 5, 6 and 7. Examination of the estimated densities and estimated errors indicates that this "fine-structure" may not be just statistical variance. On the basis of the slight information available, however, no special significance can be attached to these density variations.

Comparison with Previous Results

The crater counts and resultant statistics developed in this report differ significantly from crater statistics published by previous authors, particularly Opik, Shoemaker, and McGillem and Miller. There are two important differences. The first disagreement is in the numerical values obtained for A,B. These are contrasted in Table IV.

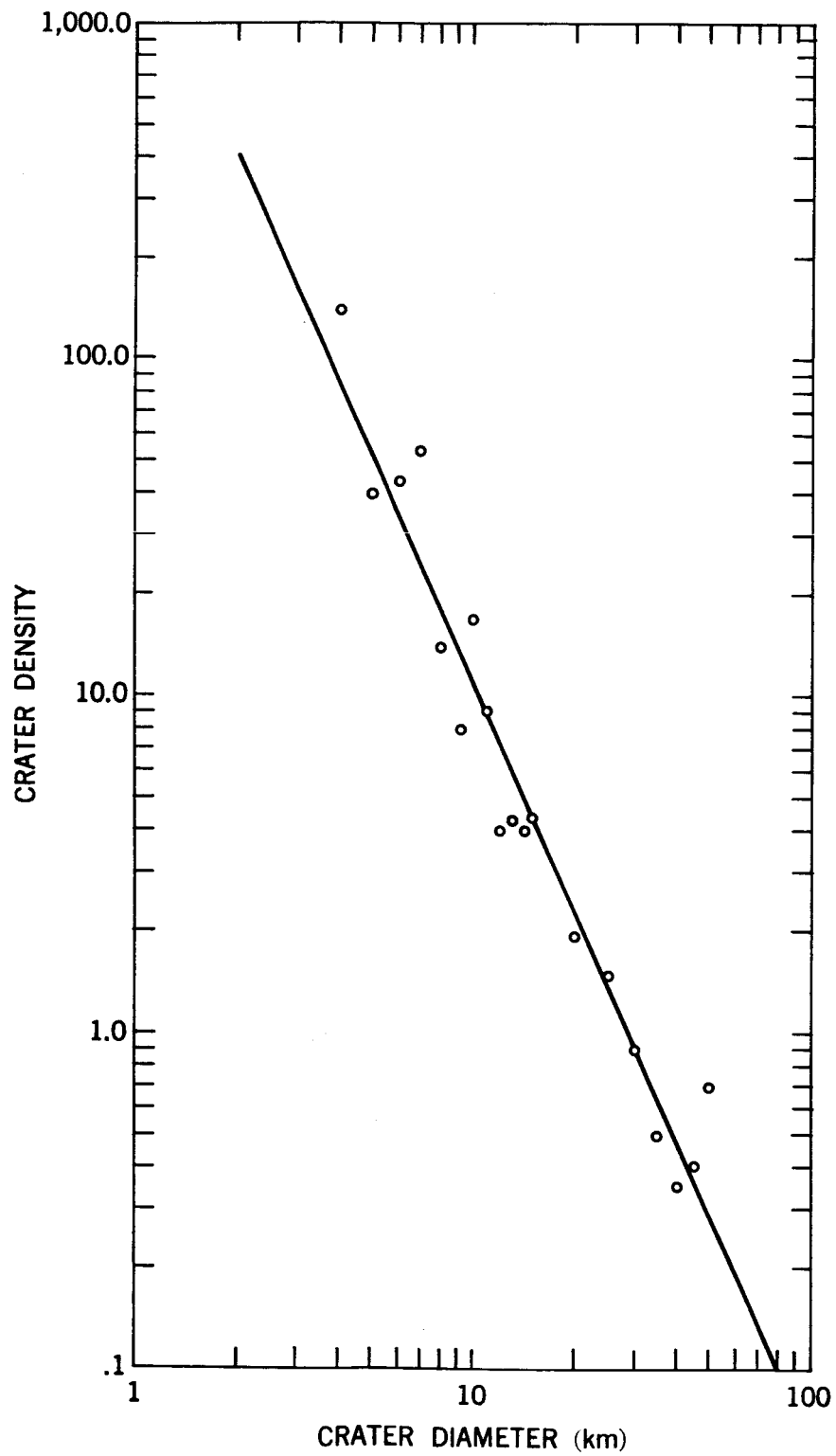
10-a

**Density Distribution Functions**

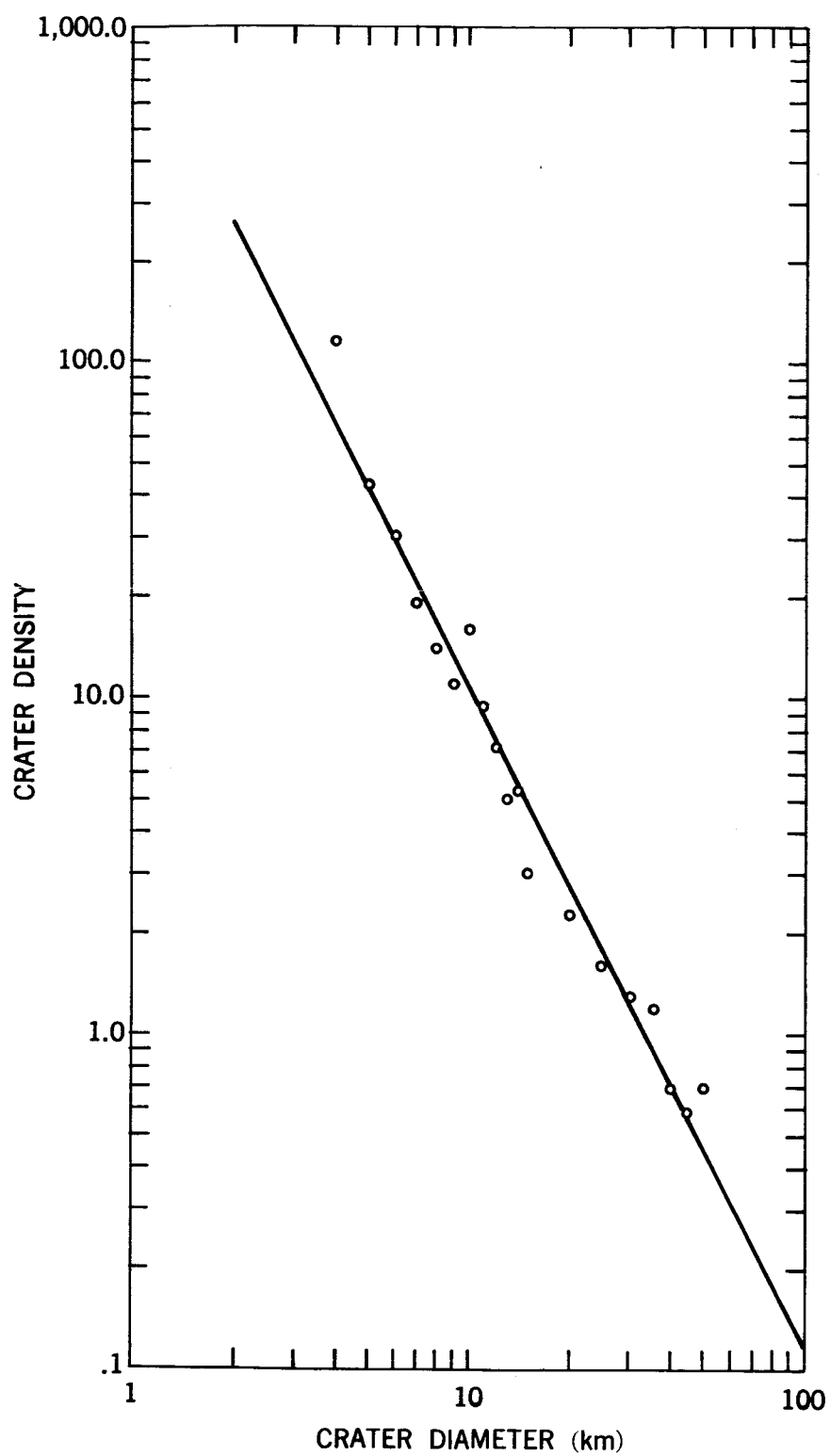
**Figure 5:      Area      C5**

**Figure 6:      Area      C6**

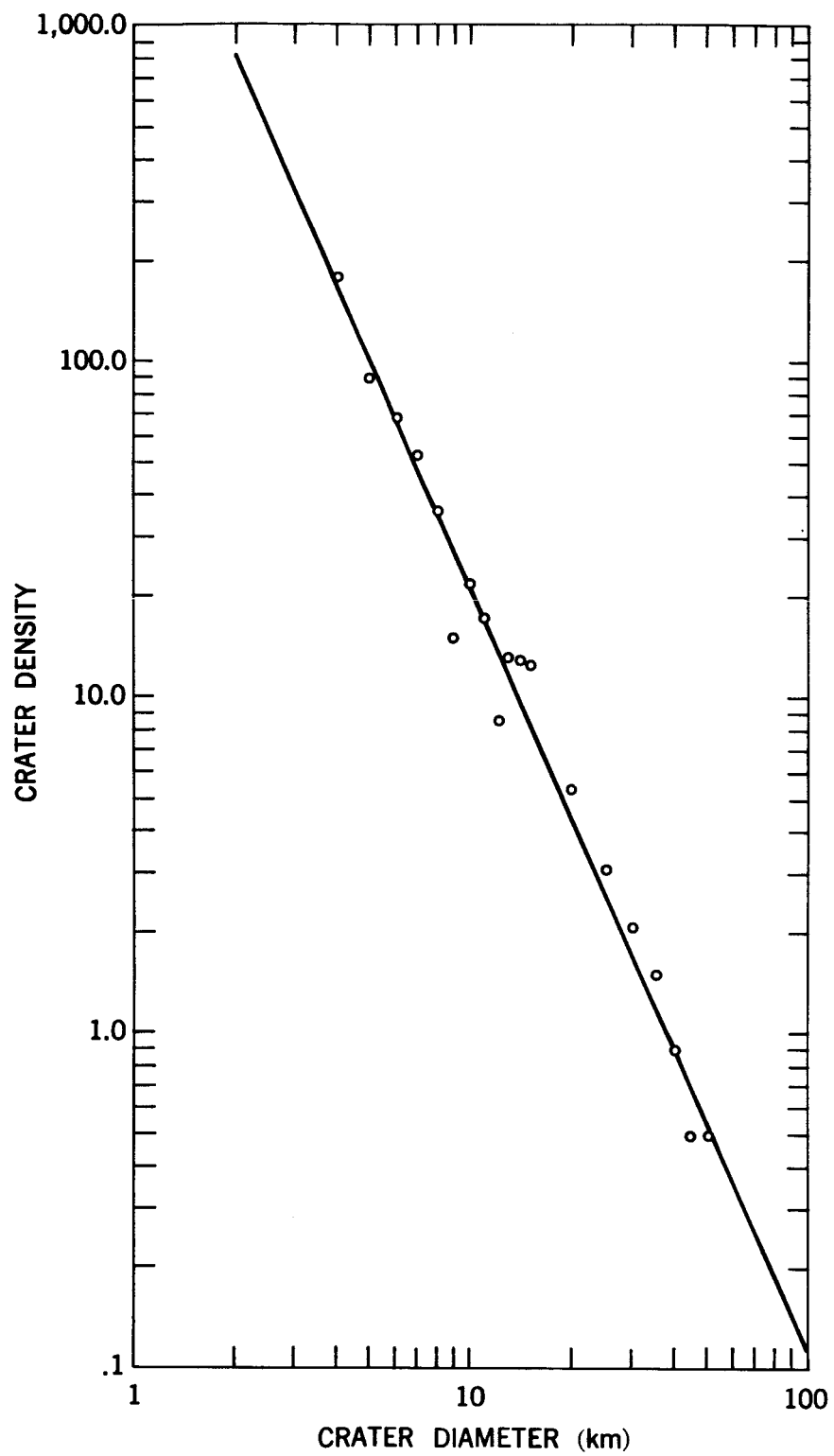
**Figure 7:      Area      C7**



Density Function—C5



Density Function—C6



Density Function—C7

TABLE IV

Authority	A	B
Opik*	1,563	-1.50
Shoemaker**	1,259	-1.60
McGillem & Miller	28,770	-1.70
Present Report:		
1 segment (avg.)	17,625	-1.53
2 seg, small dia (avg.)	9,035	-1.25

\*Opik's data fitted by Friesen.

\*\*Shoemaker's data fitted by McGillem & Miller

The second difference is in the general shape of the cumulative distribution function. The present report observes two distinct segments for the cumulative distribution function, while earlier authors either do not find this feature or do not attach significance to it.

Of the previous work, the counts developed by Opik<sup>(4)</sup> present little problem. His original data was drawn from a sample containing largely maria, or lowland regions. Much of the work was based on lunar maps, supplemented by a photograph. As with the work of Shoemaker, the difference in statistics is due to the different area of lunar surface being studied.

The work of Shoemaker<sup>(5)</sup> is concentrated exclusively in the lunar maria. The difference in Shoemaker's statistics for the maria and the present report's statistics for the highland regions is a confirmation that the apparent difference between maria and highland extends to crater statistics. There is no contradiction implied by the different statistics.

McGillem and Miller<sup>(6)</sup>, however, examined the highland regions as well as the maria. Their examination of the highland regions includes an area in common with the present work - the Maurolycus region in area C7. Moreover, their examination is based on the same photographs (the Kuiper Photographic Lunar Atlas) as is the present study. Although McGillem and Miller describe their data by a single segment least squares fitted cumulative distribution function, their data points deviated by as much as 25% from the mean values (contrasted to deviations generally less than 5% in the present work). Nor does a scattergram of their data indicate that the deviations may be due to the existence of two segments in the cumulating distribution curve; rather, the points lie about the line at random. McGillem and Miller present their crater counts as cumulative numbers for 4, 8, 16, 32 and 64 Km diameters. There is no indication of the degree of fineness to which diameters were actually measured. Nor

is their data analysis open to checking at this time since no records were kept of the craters chosen for the statistics.

In the present work diameters of all individual craters in the sample areas were measured, so that cumulative crater counts could be made at many diameters. This allows fine detail in the cumulative distribution function to be examined closely. Because the statistics available from the Boston University Catalog of Lunar Craters are strikingly consistent and of good accuracy, it will be assumed that the differences between these statistics and McGillem and Miller's statistics is due to a lack of detail and accuracy in the latter statistics.

#### Interpretation of Results

The two segment statistical description of the data is the most accurate description of the lunar crater counts. A one segment least squares fitted line for the same data yields an overestimation of the cumulative number of craters at 4 km diameter by about 30%. The one segment overestimation at diameters of 80 km is almost 50%. The two segment description, however, gives a predictive error of less than 5% throughout the range of diameters from 4 km to 80 km. Table V displays the sum of log deviations squared for both one and two segment least squares fits. The comparison shows that the two segment equation reduces the sum of log deviations by almost a factor of 10.

TABLE V

Sum of Log Deviations Squared		
Area	One Segment	Two Segment
C5	.13391	.02877
C6	.27311	.01581
C7	.04461	.00851

Any meaningful extrapolation of crater statistics below the diameter of the smallest observable craters (about 2 Km dia.) must use the small diameter equation of the two segment description. At diameters of one kilometer, the one segment equation estimates a crater density twice as great as the two segment equation, and the overestimation becomes worse as the extrapolation is pushed to smaller diameters.

There is, however, reason for expecting a one segment cumulative distribution function. Several authors (Baldwin<sup>(9)</sup>, Shoemaker<sup>(5)</sup>) have suggested that crater diameters should be proportional to the energy of the meteoroids which create them, and hence proportional to the meteoroids' mass. Hawkins<sup>(10)</sup> and others have examined meteoroid mass distributions and found no evidence of a two segment mass distribution.

The two segment equation can be rectified by adding small craters to the crater count, thus raising the small diameter counts, or by adding large craters and raising the large diameter crater counts, or by a combination of the

two. This corresponds to saying that between the time of observation, some agency has created a deficiency in either small craters, or large ones, or both very small and very large ones.

At least two agencies can cause a deficiency in small craters: (1) Loss of craters in observation due to small size. (2) Destruction of small craters due to the creation of larger ones.

At crater diameters of 1 or 2 kilometers, there is obviously a loss of craters in observation. To compensate for this effect, no craters smaller than 4 kilometers have been used in developing the crater statistics. Craters of this size should be readily visible on the photographs used. Moreover, there is no apparent drop-off in either the crater density or cumulative count as the 4 kilometer limit is approached.

A calculation has been made to determine the change in the cumulative distribution function if small craters are added to the cumulative crater count in proportion to the area occupied by the larger craters. The change in the cumulative distribution counts is small, being less than 5% of the counts. Thus, neither of these mechanisms for the loss of small craters is sufficient to rectify the two-segment curve.

It is also possible to miss very large craters in the counting procedure, for several reasons. The large craters

may have old and broken down walls, to the extent that the remaining perimeter does not meet the crater acceptance criteria. There are several formations whose type is in dispute; formations called "walled plains" may be very large craters. In the highlands regions curved "mountain chains" which lie in arc segments may well be remnants of the wall of a very ancient and large crater. It is not likely that very many such large craters would go unnoticed in any given area, but only a few would be needed to rectify the cumulative distribution curve.

The addition of 4, 7 and 6 craters with diameters greater than 80 kilometers in the unnormalized sample areas of C5, C6 and C7 respectively rectifies the cumulative distribution function. The rectified cumulative distribution functions are shown in figures 8, 9 and 10. The resultant least squares fitted parameters are given in Table VI.

TABLE VI

Area	Rectified		Two Segment Small Dia.		Craters (D > 80 Km) Added to Rectify (per 10 <sup>6</sup> km <sup>2</sup> )
	A	B	A	B	
C5	9,772	-1.2642	11,639	-1.3757	26
C6	7,692	-1.1627	7,185	-1.1768	26
C7	8,994	-1.2556	8,280	-1.2370	28

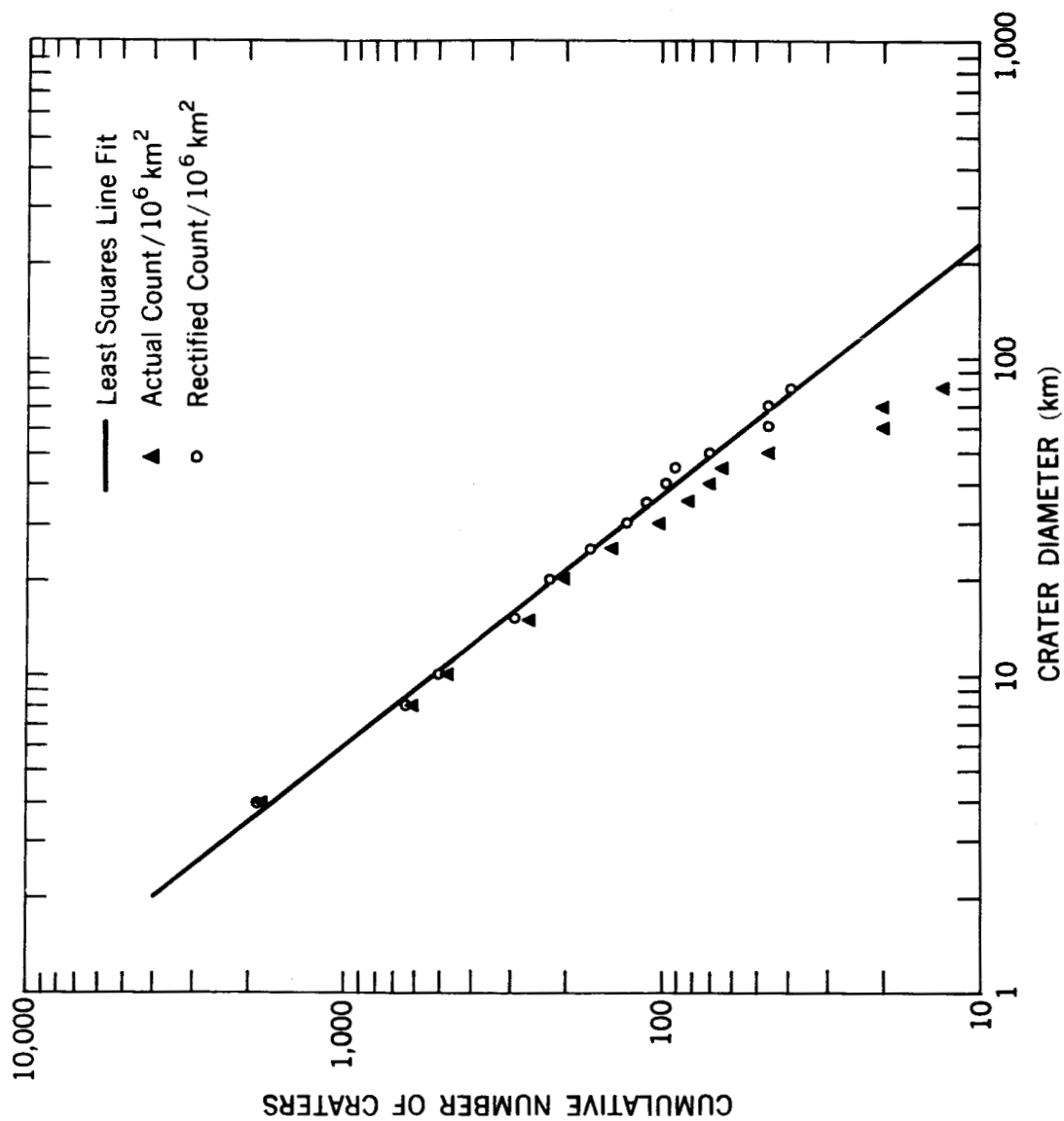
16-a

**Rectified Cumulative Distribution Functions**

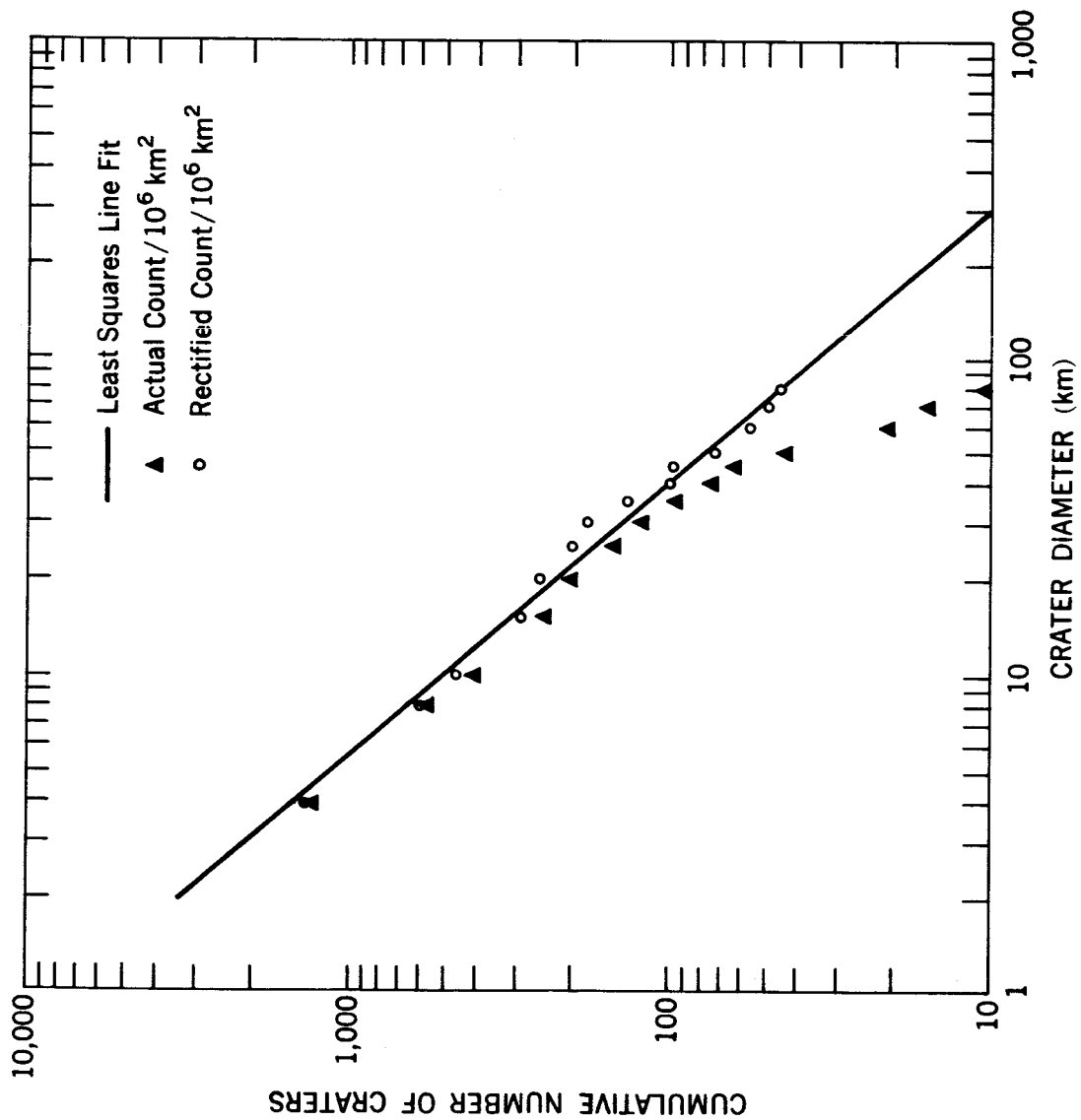
**Figure 8:      Area      C5**

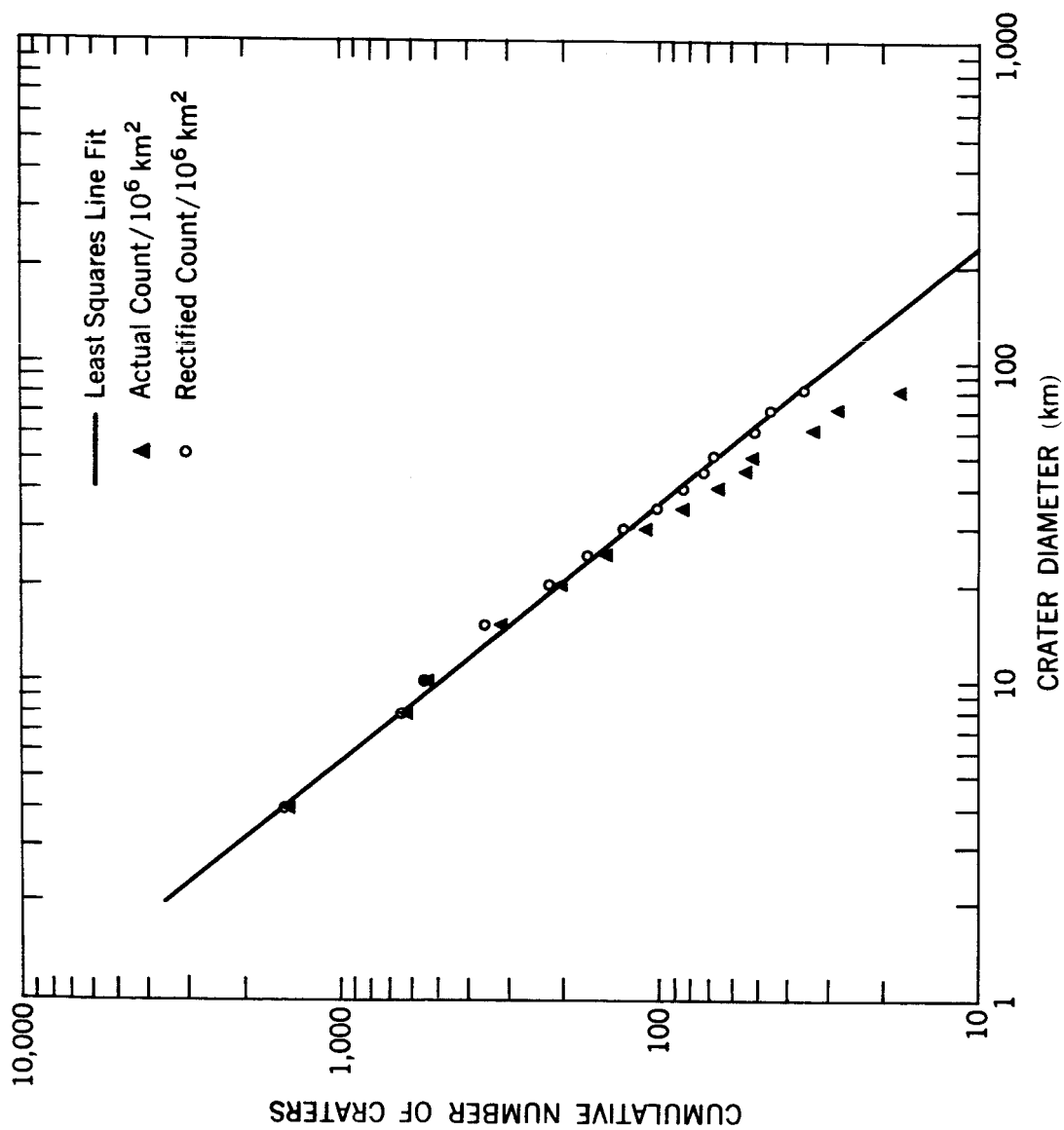
**Figure 9:      Area      C6**

**Figure 10:     Area      C7**



Rectified Cumulative Distribution Function—Area C5  
(Normalized to  $10^6 \text{ km}^2$ )





Rectified Cumulative Distribution Function Area C7  
(Normalized to  $10^6 \text{ km}^2$ )

The rectified cumulative distribution functions constants A,B are in close agreement with the values of A,B for the small diameter two segment equation. The rectified values of B also correspond closely to the values obtained for B from the density distribution function (Table III). These close agreements suggest that the rectified cumulative distribution equations of Table VI are the best estimates for a hypothetical one-segment cumulative distribution function.

Acknowledgment:

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APPENDIX A

Crater counts, normalized to  $10^6 \text{ Km}^2$ , used in this report:

<u>Diameter</u>	<u>Cumulative Number</u>		
	C5	C6	C7
80	13.06	10.27	17.86
70	19.60	15.41	26.76
60	19.60	20.54	32.75
50	45.72	41.09	50.61
45	65.32	61.63	53.39
40	71.85	71.91	65.50
35	84.91	97.59	83.36
30	104.51	133.54	110.15
25	143.70	164.36	145.88
20	202.48	215.72	202.44
15	267.80	282.49	306.64
10	467.81	477.66	518.02
8	613.98	606.06	607.33
4	1861.50	1448.40	1449.80

Based on counts in sample areas:

Section	Area
C5	$.1531 \times 10^6 \text{ Km}^2$
C6	$.1947 \times 10^6 \text{ Km}^2$
C7	$.3359 \times 10^6 \text{ Km}^2$